Closing *Wed*: HW_7A,7B (7.5, 7.7, 7.8)

Note: Exam 2 is **Thursday**!!!

Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8

The exam will roughly look like this:

- First 3 pages: 6 integrals (*ALL* types)
- 4th page: 6.5, 7.7 and/or 7.8
- 5th page: 6.4

(8.1 Arc Length is NOT on our midterm)

A Brief Exam 2 Review

Integration!

Work.

Average Value.

Trapezoid, Simpson Rules.

Improper Integrals.

1. The 4 special methods

By parts:

$$xe^{3x}, x^2\cos(5x),$$
$$\frac{\ln(x-1)}{x^2}, x\tan^{-1}(x).$$

Trig:

$$\sin^3(x)\cos(x), \cos^4(x),$$
$$\sec^3(2x)\tan^3(2x)$$

Trig sub:

$$\frac{1}{x\sqrt{x^2-9}}, \frac{1}{(4-x^2)^{3/2}},$$
$$\sqrt{x^2+6x+10}.$$

Part Frac:

$$\frac{x+2}{(x-1)(x-3)}, \frac{4x}{(x-1)^2(x-3)},$$
$$\frac{5}{x(x^2+4)}, \frac{x^2}{x+7}, \frac{x-3}{x^2+8x+20}$$

2. Substitution and Simplifying

Try $u = \sqrt{x}$, u = inside, u = e^x, u = trig.

Completing the square.

Trig facts.

Square identities, half-angle.

Triangle trick.

Random Integrals from Old Finals:

1. $\int \frac{1-x}{\sqrt{1-x^2}}\,dx$

2. $\int \frac{x^2-x+8}{x^3+4x}\,dx$

3. $\int 2x\ln(x+5)\,dx$

4. $\int \cos^3(x)\,dx$

5. $\int_0^2 \frac{1}{\sqrt{x^2+2x+4}}\,dx$

6. $\int_1^3 \frac{1}{x^2+x^3}\,dx$

7. $\int \tan^2 x \sec^4(x)\,dx$

8. $\int \frac{1}{\left(1+\sqrt{x}\right)^3}\,dx$

9. $\int \sin(x)\sqrt{\cos(x)}dx$

3. Improper Integrals:

a) Rewrite as a limit!!

b) Integrate

c) Take limit

Random Improper Integrals:

1. $\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$
2. $\int_{-3}^{\infty} xe^{-x}\,dx$
3. $\int_1^{\infty} \frac{1}{\sqrt{x}(1+x)}\,dx$

4. Trapezoid/Simpson Rules

a) Set up integral, then compute width and label tickmarks.

b) Use formula.

Approximation Example:

1. Use Simpson's Method with n = 4 subdivision to approximate the value of

$$\int_0^4 \sqrt{1+4x^4}\,dx$$

5. New Applications

a) **Average value** = $\frac{1}{b-a}\int_a^b f(x)dx$

b) **Work** = $\int_a^b (Force)(Dist)$

Step 1: Draw picture (start and end)
Label clearly.
Draw a typical subdivision.

Step 2: Find pattern for *Force* and *Dist*.

Step 3: Integrate.

Type 1: "Changing force"

Force changing as object is moved
(leaky bucket, springs, given force).

f(x) = "force formula at x"

Force = f(x), *Dist* = Δx;

$\text{Work} = \int_a^b f(x)dx$

Type 2: "Stack of books"
(chain, pumping)
Chain/Cable:

Given *k* = density = force/length
if x = 0 is labeled at the top.
then for any subdivision that makes it to the top:
Force = k Δx, *Dist* = x

$$\text{Work} = \int_a^b k \, \text{x} \, dx$$

Pumping:

Given k = density = force/volume
if top is y = b, then
Force = k(Area)Δy, *Dist* = b – y;

$$\text{Work} = \int_a^b k(\text{Area})(b - y)dy$$

You need to have an basic understanding of how we label and find pattern to be able to adapt to changes in these problems!!

Applications from old tests:

1. Find the average value of $\cos^3(x)$ on the interval 0 to $\pi/2$.

2. A spring has natural length of 30 cm from the wall. It requires 2 J of work to stretch it from 40 cm to 45cm (from the wall). How far beyond its natural length will a force of 64 N keep the spring stretched?

3. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

4. A rope is used to pull a bucket full of water up from a well that is 10 m deep. The rope has a total mass of 5 kg. The bucket has a mass of 11 kg. Find the total work done in lifting the bucket to the top (Recall: Accel. due to gravity is 9.8 m/s^2)

5. A well is in the shape of a cylinder of radius 1 meter and depth 8 meters. It is half full of water. Find the word to pump all the water to the top. (Recall: Water weighs 9800 N/m^3)

6. The portion of the graph $y = x^2 / 9$ between $x = 0$ and $x = 3$ is rotated about the y-axis to form a container. The container is full of a liquid that has density 100 lbs/ft^3. Find the work required to pump all the liquid to the top of the container.